

Snow II: Snowmelt and energy balance

The are three basic snowmelt phases

- 1) Warming phase: Absorbed energy raises the average snowpack temperature to a point at which the snowpack is isothermal (no vertical temperature gradient) at 0°C.
- 2) Ripening phase: Absorbed energy is used to melt snow, but the meltwater is retained in the snowpack in pore spaces by surface tension forces. As the end of this phase, the snowpack cannot retain any more liquid water and is said to be “ripe”.
- 3) Output phase: Further absorption of energy produces water output, which then appears as runoff, infiltration or evaporation.

This is a very idealized sequence. For example, melting typically occurs at the surface of the snowpack at a time prior to the ripening phase. Meltwater may percolate into deeper layers where the snow temperature is below 0°C and refreeze, forming ice layers and lenses; latent heat release then warms the snowpack. There is also (typically) a diurnal freeze/thaw cycle.

Warming phase

Warming phase: Must add energy to raise the temperature to 0°C. The required energy represents the “cold content” (Q_{cc}) of the snowpack. This energy total, in $J\ m^{-2}$ (a snowpack 1 meter on a side), is:

$$Q_{cc} = -c_i \cdot \rho_w \cdot h_m \cdot (T_s - T_m)$$

Where c_i is the heat capacity of ice ($2102\ J\ kg^{-1}\ K^{-1}$), T_s is the average temperature of the snowpack, T_m is the melting point of ice ($0^\circ C$), ρ_w is the density of water (approximately $1000\ kg\ m^{-3}$), and h_m is the snowpack water equivalent in meters.

For a snowpack water equivalent of 0.29 m and an average snowpack temperature of $-9^\circ C$ (Example 5-2 in Dingman's book), the cold content is $5.49\ MJ\ m^{-2}$.

What if we do not have snowpack water equivalent but rather snow density and depth? Then we need to replace ρ_w with the snow density ρ_s and h_m with snow depth h_s .

$$\rho_w \cdot h_m = \rho_s \cdot h_s \text{ (units } kg\ m^{-2}\text{)}$$

Ripening phase

From empirical studies, the maximum volumetric water content (θ_{ret}) that a snowpack can retain against gravity (for ripe snow) is:

$$\theta_{ret} = -0.0745 \cdot (\rho_s / \rho_w) + 0.000267 \cdot (\rho_s / \rho_w)^2$$

For a typical ripe snowpack of density $\rho_s = 500 \text{ kg m}^{-3}$, $\theta_{ret} = 0.03$

Snowpack density can be written as:

$$\rho_s = (1 - \Phi) \cdot \rho_i + \theta \cdot \rho_w$$

ρ_i = ice density = 917 kg m^{-3}

θ = liquid water content, as the ratio of liquid water volume to total snowpack volume

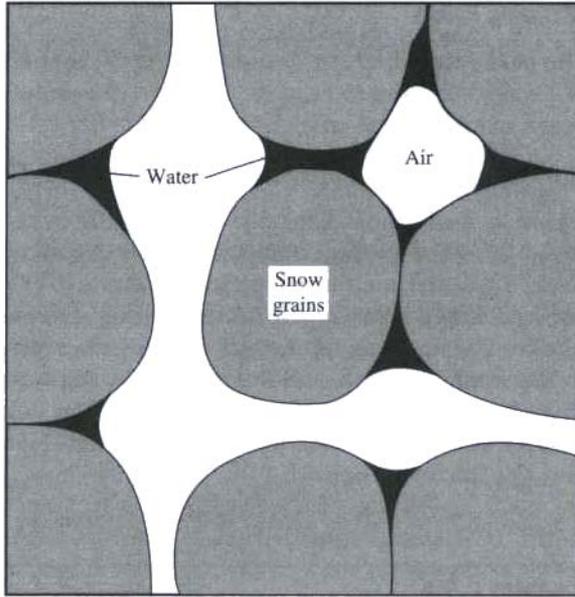
Φ = porosity, the ratio of pore volume to total snowpack volume

With $\rho_s = 500 \text{ kg m}^{-3}$ and $\theta_{ret} = 0.03$, and rearranging, $\Phi = 0.49$

$\theta_{ret} / \Phi = 0.061$ = proportion of pore space filled with water at the end of the ripening phase. Key: Little of the pore space (6% in this example) is water!

Net energy Q_{m2} in J m^{-2} require to complete the ripening phase is:

$$Q_{m2} = \theta_{ret} \cdot h_s \cdot \rho_w \cdot \lambda_f, \text{ where } \lambda_f \text{ is the latent heat of fusion (} 0.334 \text{ MJ kg}^{-1}\text{)}$$



Dingman 2002, Figure 5-19

Idealized depiction of snow grains, water retained by surface tension and continuous pores filled with air.

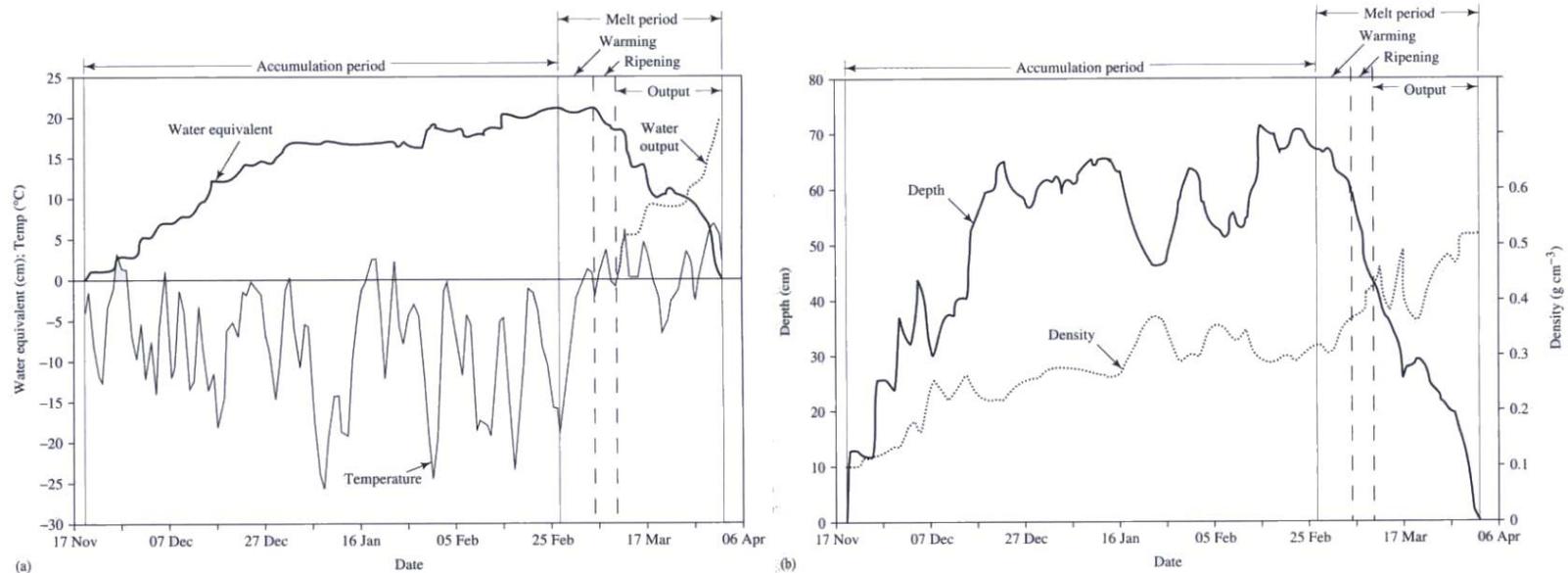
Output phase

Once the snowpack is ripe, further energy input results in meltwater that percolates downward to become water output. The energy Q_{m3} in J m^{-2} required to melt the snow remaining at the end of the ripening phase is:

$$Q_{m3} = (h_m - h_{wret}) \cdot \rho_w \cdot \lambda_f$$

Where $h_{wret} = \theta_{ret} \cdot h_s$ is the liquid-water retaining capacity of the snowpack and h_m (as we recall) is the liquid water equivalent of the snowpack.

Time series from Sleepers River Research Watershed (VT)



Dingman 2002, Figure 5-24

The increase in snow depth and snowpack water equivalent (SWE) through the accumulation season is accompanied by an increase in density due to snow metamorphism. The accumulation period ended Feb 26 and SWE began to decline on 4 March. Snowpack density continued to increase through the snowpack warming phase, ripening phase and water output phases of the snowmelt process, reaching 520 kg m⁻³ just before the melting process was complete.

The energy balance

The snowmelt process requires absorption of energy Q of an amount equal to the sum of the energy absorption associated with the three snowmelt phases:

$$Q = Q_{cc} + Q_{m2} + Q_{m3}$$

$$Q = -c_i \cdot \rho_w \cdot h_m \cdot (T_s - T_m) + \theta_{ret} \cdot h_s \cdot \rho_w \cdot \lambda_f + (h_m - h_{wret}) \cdot \rho_w \cdot \lambda_f$$

Recall that Q has units of J m^{-2} . Dividing each term through by time yields a time rate of change of energy storage $(\partial Q / \partial t) = \text{J m}^{-2} \text{s}^{-1} = \text{W m}^{-2}$ of the water in the 1x1 meter column representing the original snowpack.

From energy conservation:

$$\partial Q / \partial t = \text{energy flux in} - \text{energy flux out} \quad (\text{W m}^{-2}) \text{ from all processes.}$$

The energy budget

$$\partial Q/\partial t = K + L + H + LE + R + G$$

$\partial Q/\partial t$ = net energy flux into snowpack (change in storage)

K = net shortwave radiation flux

L = net longwave radiation flux

H = turbulent sensible heat flux

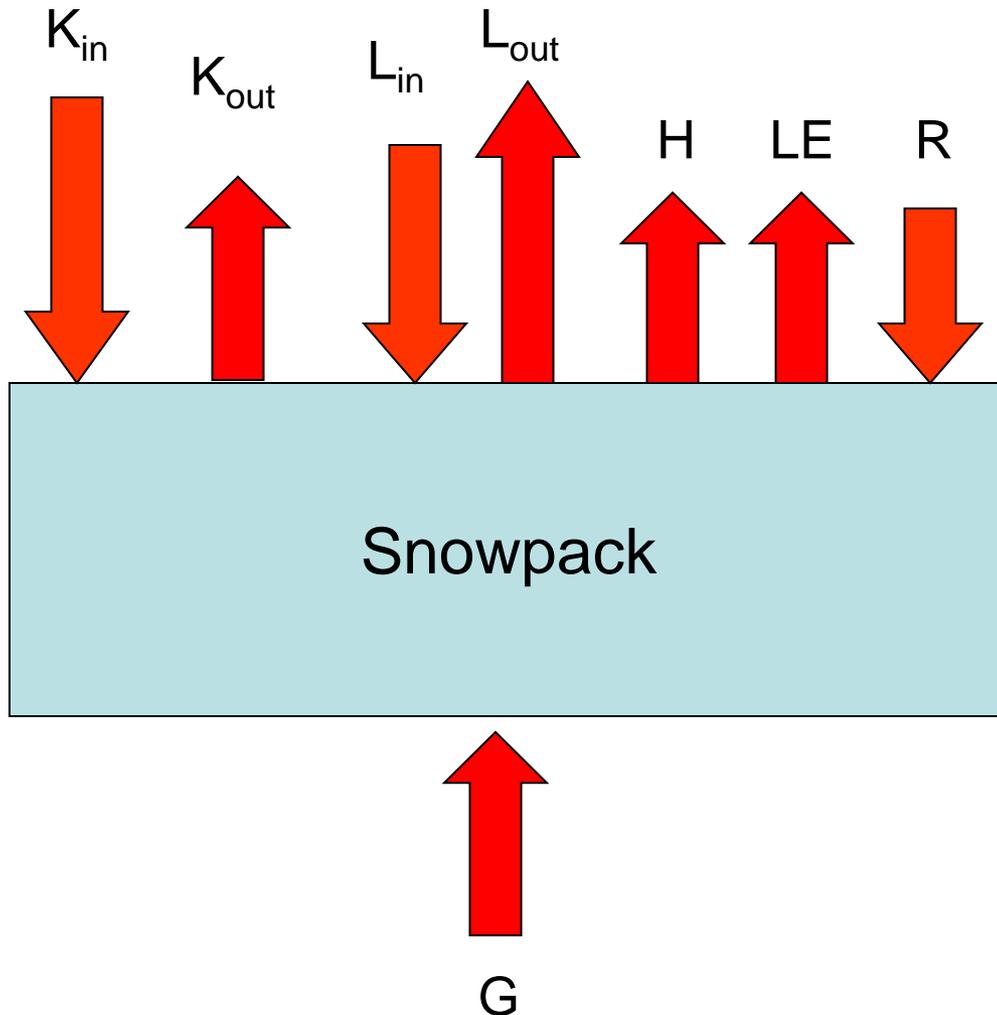
LE = turbulent latent heat flux heat

R = heat input from rain

G = conductive exchange of sensible with the ground

All fluxes are positive when directed into the snowpack (i.e., they are an energy *source* to the snowpack).

The energy budget



$$K = K_{in} - K_{out}$$

$$K = K_{in} (1 - \alpha), \alpha = \text{albedo}$$

K is either positive or zero

$$L = L_{in} - L_{out}$$

$$L = \sigma \epsilon_a T_a^4 - \sigma \epsilon_s T_s^4$$

ϵ_a, ϵ_s = emissivity of surface and atmosphere, respectively

T_a, T_s = temperature of the air and surface, respectively

σ = Stefan-Boltzmann constant

L is usually negative

H, LE , typically both negative, but by no means always

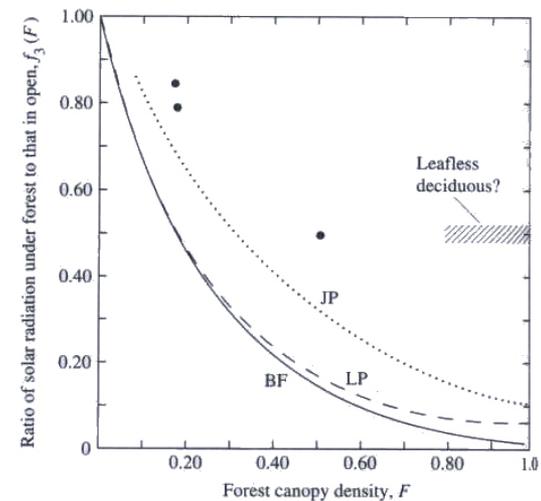
G is usually positive

R is always positive

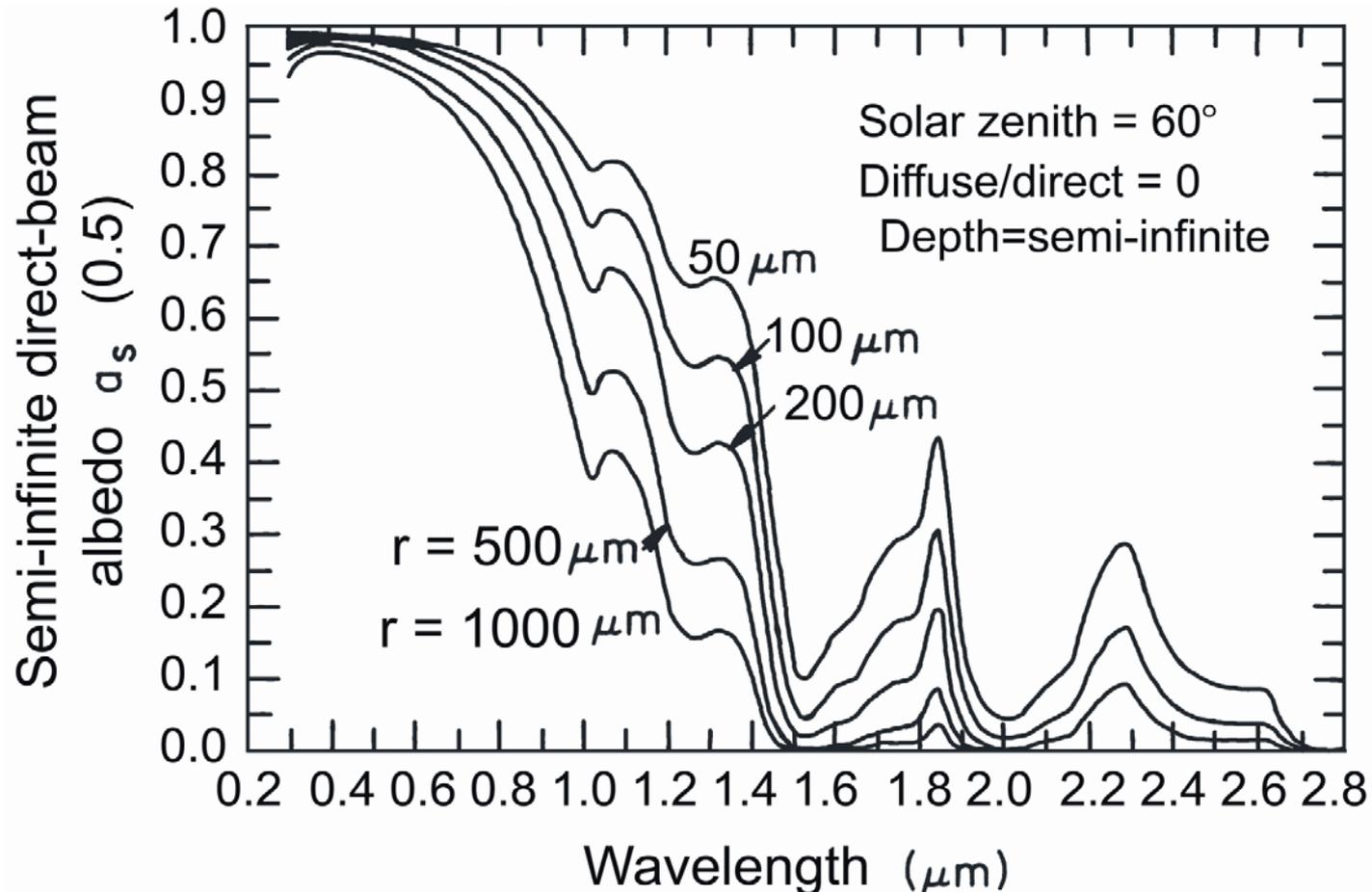
Controls on net shortwave radiation K

- Biggest in summer, smallest in winter (small or zero in high latitudes in winter)
- Depends strongly in surface albedo. Albedo of fresh snow is about 0.8, albedo of forest might be 0.15-0.20. However, snow albedo is quite variable and drops as snow undergoes metamorphosis.
- Depends on slope and aspect, more on south-facing slopes than north-facing slopes
- Reduced under cloud cover, primarily due to high cloud albedo, and to a lesser degree by cloud absorption; both are related to cloud thickness.
- Reduced by “clear sky” scattering and absorption
- (atmospheric gases, aerosols)
- Reduced under forest canopies

At right: Ratio of incident solar radiation to the surface under various types of forest canopies to that received in the open for four forest types, BF = balsam fir, JP = jack pine, LP = lodgepole pine, circles = open boreal spruce forest.

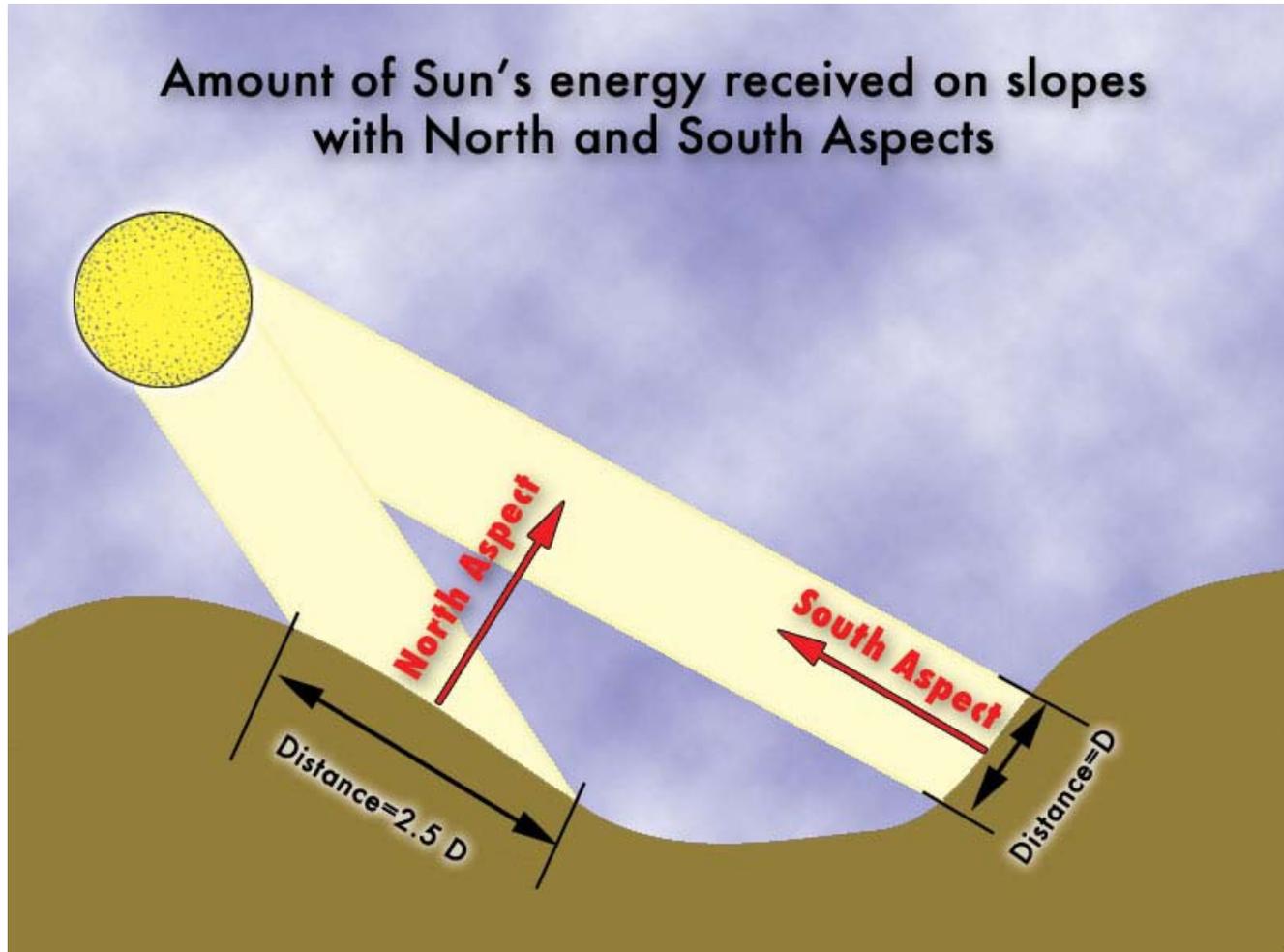


Spectral reflectance



Direct beam spectral reflectance for a semi-infinite snowpack as a function of wavelength for grain radii from 50 to 1000 μm and for a solar zenith angle of 60° [from Wiscombe and Warren, 1980, by permission of AMS]. The key points are that spectral reflectance depends on both grain size and wavelength. Albedo is the integrated spectral reflectance over the visible wavelengths.

Effects of slope and aspect on incoming shortwave radiation



Controls on net longwave radiation L

Recall that:

$$L = L_{in} - L_{out}$$
$$L = \sigma \cdot \epsilon_a \cdot T_a^4 - \sigma \cdot \epsilon_s \cdot T_s^4$$

L_{out} : Most surfaces behave approximately as blackbodies in the infrared ($\epsilon_s \approx 1$), so if you know the surface temperature, you know L_{out} . The surface temperature is the radiating skin temperature, not the air temperature.

L_{in} : This is much more complicated. While L_{in} depends on T_a , the near-surface air temperature, it is strongly dependent on the atmospheric emissivity, ϵ_a . The emissivity increases with increasing water vapor content (water vapor is a greenhouse gas). It is also higher under cloudy skies than under clear skies, as cloud bases act like blackbody emitters. Trees are also very nearly blackbodies, such that L_{in} is higher under a forest canopy than an adjacent open field.

In most cases, outgoing longwave radiation exceeds incoming longwave radiation i.e., L is negative. However, it becomes less negative and can even be positive under heavy cloud cover, forest cover, and inversion conditions.

Turbulent sensible and latent heat fluxes

Through diffusion, there will be a vertical exchange of sensible heat H (heat as measured by a thermometer) between the surface and the overlying atmosphere, the direction of which is determined by the sign of the vertical temperature gradient (upward if temperature decreases with height, downward if temperature increases with height) and magnitude which is proportional to the strength of the gradient and factors governing the efficiency of the diffusion process. Similarly, the sign of the vertical latent heat flux is determined by the vertical gradient in water vapor, with magnitude proportional to the strength of the gradient and the efficiency of the vapor diffusion process:

$$H = -C_H \cdot \rho_a \cdot c_a \cdot \partial T / \partial z$$

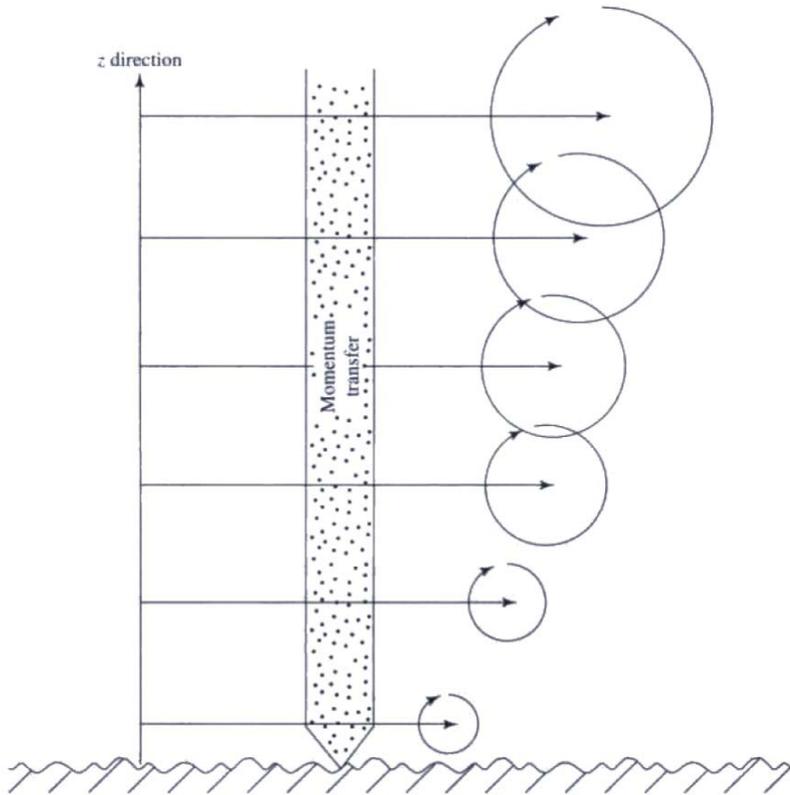
Where ρ_a is air density, c_a is the specific heat of dry air, T is temperature, z is height and C_H is the “efficiency” coefficient for sensible heat transfer.

$$LE = C_E \cdot \lambda_v \cdot \partial p_v / \partial z$$

Where λ_v is the latent heat of vaporization, p_v is absolute humidity and C_E is “efficiency” coefficient for latent heat transfer. Absolute humidity is the ratio of the mass of water vapor to the volume occupied by a mixture of moist and dry air, i.e., the density of the water vapor component.

Diffusion: The process of mixing fluid properties by both molecular and turbulent motions. Diffusion is a consequence of *concentration gradients*.

Turbulent latent and sensible heat fluxes (cont)



Dingman 2002, Figure D-9

Molecular diffusion (conduction) is inefficient. The process is instead dominated by turbulence, known as *eddy diffusion*. A horizontal wind blows across a surface. There is friction with the surface, resulting in a reduction of the horizontal wind as the surface is approached, attended by a turbulent flow with a highly variable vertical component of the wind. This friction and turbulence results in a net downward transfer of momentum. If temperature decreases with height (the usual situation), then when the wind is directed upwards, it carries warm air away from the surface to higher levels. When the wind is downward, it carries cool air towards the surface. The net result is an upward transfer of sensible heat. Similarly, if humidity decreases with height (the usual situation), upward winds carry moist air away from the surface and downward winds carry drier air towards the surface, resulting in net upward transport of water vapor. If the temperature and humidity gradients are reversed, the respective turbulent flux is upwards. The stronger the winds, the stronger the turbulence and momentum transfer, and the stronger the turbulent sensible and latent heat fluxes for a given temperature or humidity gradient. If there is no gradient, there is no flux.

Turbulent sensible and latent heat fluxes (cont)

Recall that the turbulent sensible and latent heat fluxes can be expressed, respectively, as

$$H = -C_H \cdot \rho_a \cdot c_a \cdot \partial T / \partial z$$
$$LE = C_E \cdot \lambda_v \cdot \partial p_v / \partial z$$

The “efficiency coefficients” depend on the characteristics of the turbulence. The most accurate measurements of H and LE are obtained by the eddy fluctuation method, in which H and LE are proportional to the mean of the product of departures from the time means of the vertical wind and temperature, and the vertical wind and humidity, respectively:

$$H = \rho_a \cdot c_a \cdot \langle w' T' \rangle$$
$$LE = \lambda_v \cdot \langle w' p_v' \rangle$$

Where w is the vertical wind, the primes indicate departures from the time mean and the angle brackets denote the time mean. The direct approach obviates the need for determining coefficients and is widely used (although instrumentation is expensive and difficult to maintain).

So-called bulk or profile methods use measured profiles of temperature and humidity (at two or more levels) along with vertical profiles of the horizontal wind. The vertical wind profile contains information regarding the nature of the eddies under a number of assumptions, a key one being that the vertical wind profile is logarithmic. See Dingman (2002) for further discussion.

Heat input by rain (R)

There are two situations

1) Rain falls on a snowpack that is at the freezing point ($T_s = T_m = 0^\circ\text{C}$)

$$R = \rho_w \cdot c_w \cdot r \cdot (T_r - T_m)$$

Where c_w is the heat capacity of water ($4.19 \times 10^{-3} \text{ MJ kg}^{-1} \text{ K}^{-1}$),
 r is the rainfall rate (m s^{-1}) and T_s is the temperature of the rain.

Rain cools to the freezing point, giving up sensible heat, and the heat given up is used in melting.

2) Rain falls on a snowpack below the freezing point

$$R = \rho_w \cdot c_w \cdot r \cdot (T_r - T_m) + \rho_w \cdot \lambda_r \cdot r$$

Rain is cooled to the freezing point, giving up sensible heat, and then the water freezes, giving up latent heat. This can be a very effective heating process.

Conductive exchange of sensible heat with ground (G)

Temperatures in the soil underlying the snowpack generally increase downward. As such, in these cases, heat is conducted upwards to the base of the snowpack at a rate G given as:

$$G = k_G \cdot (dT_G/dz)$$

k_G = thermal conductivity of the soil ($W m^{-1} K^{-1}$)

T_G = soil temperature

z = distance below the ground surface

Hence the heat transfer depends on the temperature gradient in the soil (dT_G/dz) and how good a conductor the soil is (k_G)

Values of k_G of soils and other substances vary widely, taken from Oke (1987):

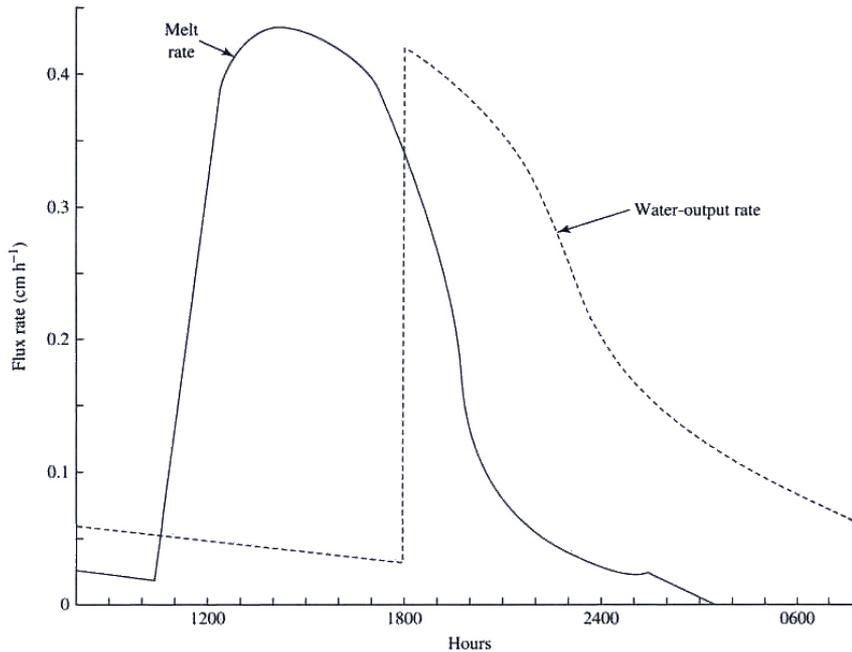
Sandy soil, 40% pore space: 0.03, dry, 2.20, saturated

Peat soil, 80% pore space: 0.06 dry, 0.50 saturated.

Snow: fresh, 0.06, old, 0.42

Water: 0.57 (at 4°C, when still)

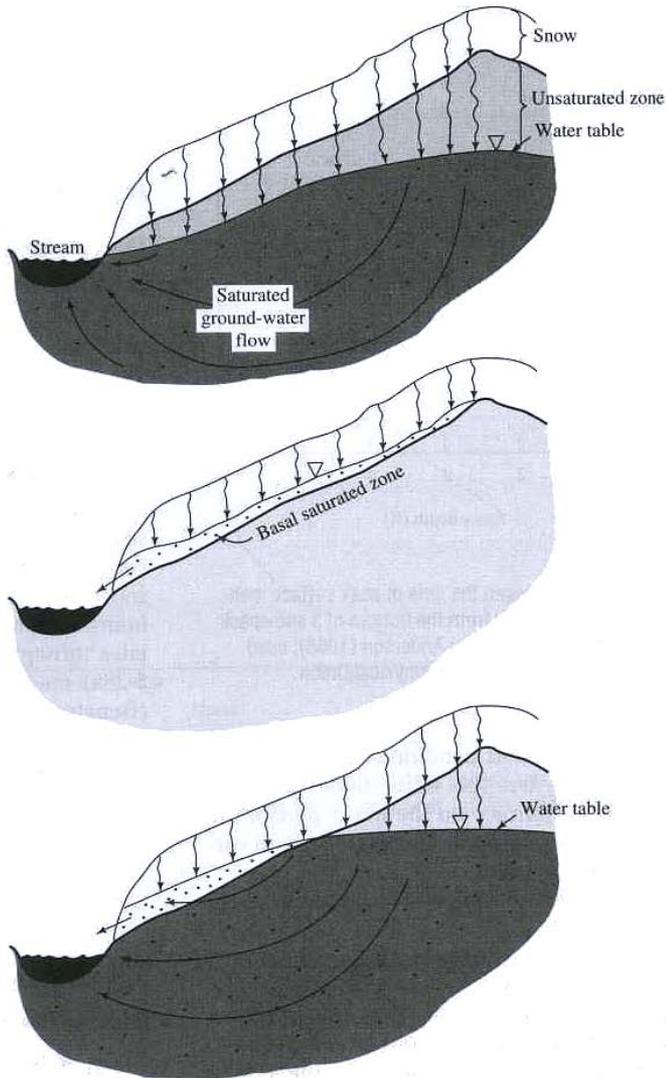
Water output from the snowpack



Dingman 2002, Figure 5-26

The surface melt rate typically exhibits a strong diurnal cycle, peaking in early afternoon. The meltwater produced at the surface then percolates downwards through the snowpack. Because the speed of the percolation increases as the melt rate increases, water produced during the peak melt period overtakes water produced earlier in the day, such that the diurnal melt wave at successively deeper depths develops a sharp wave front. This sharp wave front is clearly seen at left in the output rate of meltwater through the bottom of the snowpack (in this case a 101 cm deep snowpack). The time lag between the timing of peak surface melting and peak water output through the bottom of the snowpack increases with the depth of the snowpack.

Snowmelt runoff generation modes



Snowmelt runoff generation can take several modes. In the first panel, the top of the saturated zone in the soil (the water table) is deep, so percolating meltwater infiltrates to raise the water table and induce increased groundwater flow to the stream. In the second panel, the water table is at the soil surface, or the soil surface is impermeable (the ground may be frozen) so percolating meltwater forms a basal saturated zone through which water migrates to the stream. In the bottom panel, the lower portion of the water table has risen above the ground surface into the snowpack. Water in the upper part of the slope moves as in the top panel, and the water in the lower part of the slope moves as in the middle panel.

A few notes on snowmelt modeling

Q: Why model snowmelt?

A: There are practical applications for water management. Snowmelt models represent a component of hydrologic models that are key tools for predicting seasonal streamflows in areas like Colorado. On the larger scale, If we want to get the hydrologic cycle right in coupled global climate models, we need to get snowmelt processes right.

Basic model types:

Energy balance models: These are described as “physically based” because the energy balance is a fundamental principal and because they use equations describing the physics of processes in each component of the energy balance. Complex Land Surface Models (LSMs) formulate snowmelt from the energy balance.

Conceptual models: These make use of empirical relationships. A good example of a simple conceptual models is estimating snowmelt with a temperature index approach, in which snowmelt is a linear function of daily air temperature or melting degree days. The approach works because the air temperature tends to be well correlated with the energy balance. More complex approaches may describe variability in melt as a function of land type, slope and aspect and other factors.