Ice model for lakes in Finland

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1 Introduction

A thermodynamical lake ice model has been developed for Watershed Simulation and Forecasting System (WSFS) (Vehviläinen et al., 2005). It is based of Stefan's law that takes account the snow on top of ice, forms slush and snow ice. Also the thickening ice cover is taken into account when calculating the thermal conductivity. Snow thickness and snow density are taken from WSFS's snow model for open areas. The model is simplified and adjusted version of model developed by Saloranta (2000).

It does not take account radiative fluxes, wind or bottom flux. The air temperature is assumed to be the same as the surface temperature of ice.

2 Ice formation

The draft (part that is under the water) of ice is calculated:

$$h_d = \frac{\rho_s h_s + \rho_i h_{ice} + \rho_{si} h_{si} + \rho_{sl} h_{slush}}{\rho_w} \tag{1}$$

The snow depth and density are from snow model of the WSFS. The snow depth is simulated on open field and thus it has to be adjusted. In the ice model snow depth difference between days is calculated and added to the snow thickness on the ice. The snow on top of the ice starts to accumulate after the ice has formed. The heat conductivity of the snow is calculated with following equation:

$$k_s = 0.0285 \rho_s^2, \tag{2}$$

where minimum value of ρ_s is 0.1 g/cm^3.

Table 1: Constants of the model. Most of the constants are from Saloranta (2000)

Constant	value
ρ_i [g/cm^3]	0.917
ρ_{sl} [g/cm^3]	0.920
ρ_{si} [g/cm^3]	0.875
ρ_w [g/cm^3]	0.9998395
k_i [W/(C*cm)]	0.0224
k_{si} [W/(C*cm)]	$0.5k_i$
k_{sl} [W/(C*cm)]	0.00561
L [J/g]	333
Δt [s]	60

2.1 Snow ice

The snow ice forms from slush above the black ice. Snow ice can form when $h_d > h_i$ as the water floods the snowpack on ice. In the model the snow ice forms from the surface of existing snow ice or black ice. Only snow pack and slush layer on top insulates (the k_s term) the slush layer from the air temperature. Equation for forming the snow ice from slush:

$$\frac{dh_{si}}{dt} = \frac{-k_s T_s}{\left(h_s + \frac{k_s}{k_w} h_{sl}\right) \rho_{si} L} \frac{1}{1 - \frac{\rho_s}{\rho_{si}}} \tag{3}$$

Numerical differentiation with Eulerian scheme (as h_{si} is not inside the equation):

$$h_{si}^{t+1} = \frac{-k_s^t T_a^t}{h_s^t + k_s^t / k_w h_{sl}^t} \frac{1}{\left(1 - \rho_s^t / \rho_{si}\right) \rho_{si} L}, \tag{4}$$

where T_a^t is the temperature of the air in current time step. $t + 1$ is the next time step.

2.2 Black ice

The black ice is formed with the following Stefans equation:

$$\frac{dh_i}{dt} = \frac{-k_i T_s}{h_i + \frac{k_i}{k_s} h_s + \frac{k_i}{k_{si}} h_{si}} \frac{1}{\rho_i L}, \tag{5}$$

where h_i is the ice thickness, h_s the snow thickness and h_{si} the snow ice thickness. The above equation is discretized with Runge-Kutta 4^{th} order differentiate scheme:

$$
\begin{aligned}
k_1^t &= \Delta t \frac{-k_i T_a^t}{\left(h_i^t + k_i/k_s h_s^t + 2h_{si}^t\right)\rho_i L} \\
k_2^t &= \Delta t \frac{-k_i T_a^t}{\left(h_i^t k_1^t/2 + k_i/k_s^t h_s^t + 2h_{si}^t\right)\rho_i L} \\
k_3^t &= \Delta t \frac{-k_i T_a^t}{\left(h_i^t k_2^t/2 + k_i/k_s^t h_s^t + 2h_{si}^t\right)\rho_i L} \\
k_4^t &= \Delta t \frac{-k_i T_a^t}{\left(h_i^t k_3^t + k_i/k_s^t h_s^t + 2h_{si}^t\right)\rho_i L} \\
h_i^{t+1} &= h_i^t + k_1^t/6 + k_2^t/3 + k_3^t/3 + k_4^t/6
\end{aligned}
\tag{6}
$$

where T_a^t is the temperature of the air in current time step. $t+1$ is the next time step.

3 Ice melt

Ice melting is modeled in a rather simple way. When the snow has melted the model starts to melt the snow ice first. When the snow ice has melted black ice is melted. Both the snow ice and the black ice are melted with the following degree-day formula:

$$h_{si}^{t+1} = h_{si}^t - m_{si} T_a^t \tag{7}$$

$$h_i^{t+1} = h_i^t - m_i T_a^t, \tag{8}$$

where m_{si} and m_i are constants derived from calibration of ice melt against observations (usually $m_{si} > m_i$). The calibration of these parameters has to be included into the model.

4 Structure of the model

4.1 Files

These are the files that are modified during the construction of the ice thickness model.
`hice.f90`
The subprocess for the calculation of ice thickness and everything else. This is called from valu.for. The indexes of the tables are described here.
`hice_module.f90`
The module for exchanging tables and variables between valu.for and hice.f90.

`valu.for`

The tables are allocated in valu.for and subprocess hice_laskenta (hice.f90) is called.

`hicehav.for`

The subroutine for reading the observations of ice thickness. Similar to water temperature.

`model.inc`

The number of observation sites are defined here.

`ihasema.inc`

The numbers of observation points in the model to the specific obs. sites are defined here.

`ihlisays.inc`

The lake ids for the observation site are defined here.

5 Results

Two examples are shown here lakes 14.932.1.001 and 59.912.1.001 Lammasjärvi. The total ice thickness is plotted in blue color in the figures.

The total thickness of ice seems to work fairly well, however, there are few years in both of the modelled sites where the modelled ice thickness is less than the observed. At the same time the density of snow has varied highly and been quite low taking in account the time of the year. This suggests that the low densities from the snow model affect significantly to the total ice thickness. This comes from the fact that the thermal conductivity of the snow is proportional to the second power of snow thickness. In result of this the ice thickness is sensitive to the density and thickness of the snow pack.

5.1 14

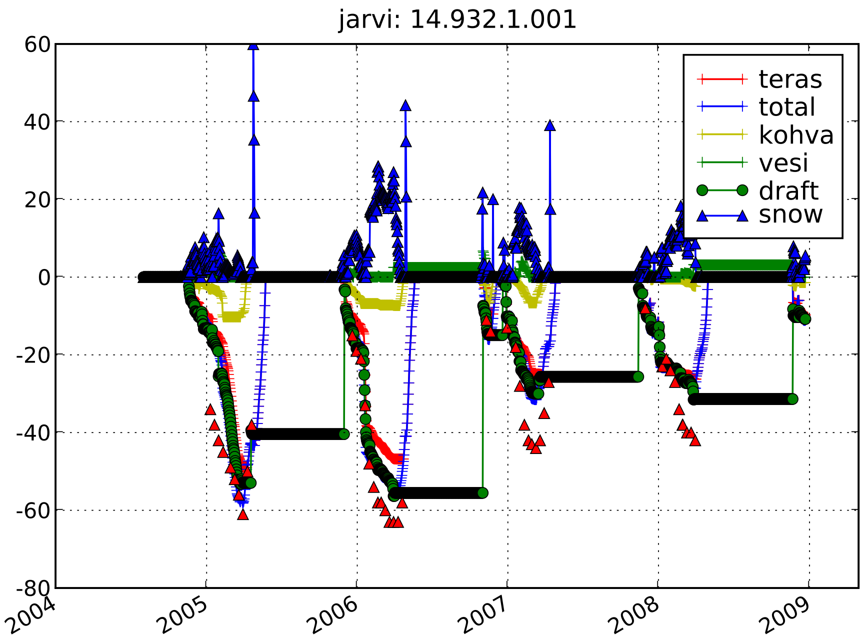

Figure 1: In the winters of 2006-2007 and 2007-2008 ice thickness from the model is less than in the observations. This seems to be mainly due the low snow densities resulting low heat conductivity of snow. See Figure 2.

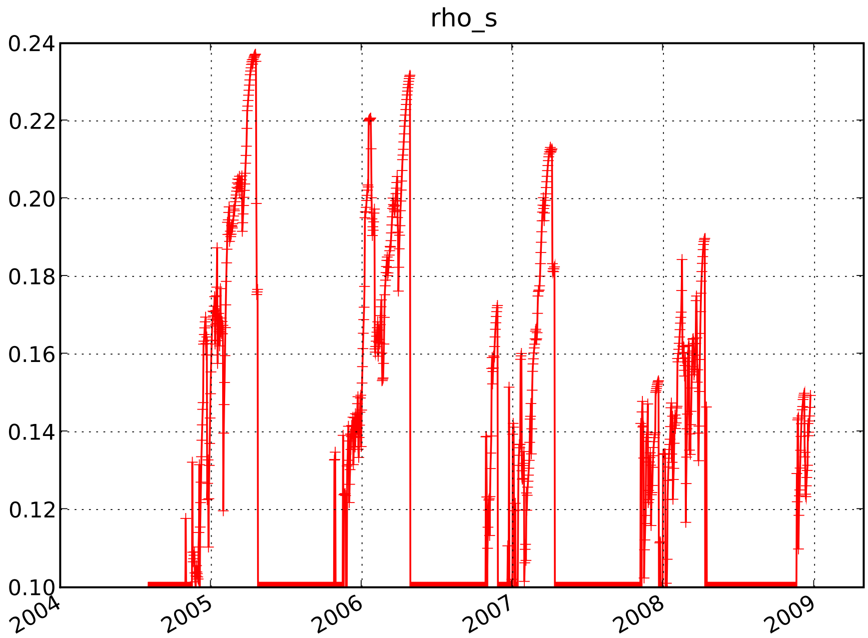

Figure 2: The densities of snow for lake 14.932.1.001. In the winters of 2006-2007 and 2007-2008 the density has high variation.

5.2 59

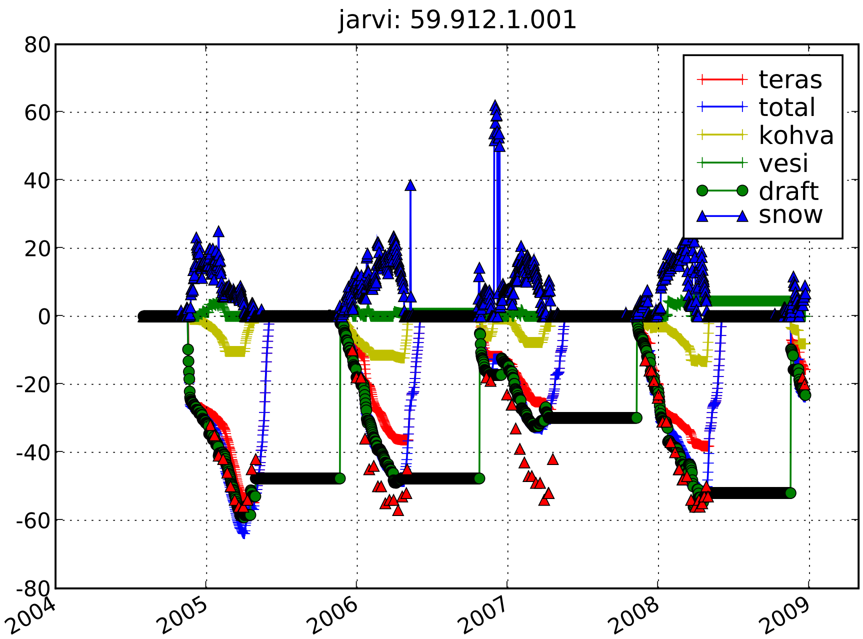

Figure 3: In winter 2006-2007 the modelled ice thickness (total) is lower than the observed. This mainly seems to be due the low densities of snow. See Figure 4.

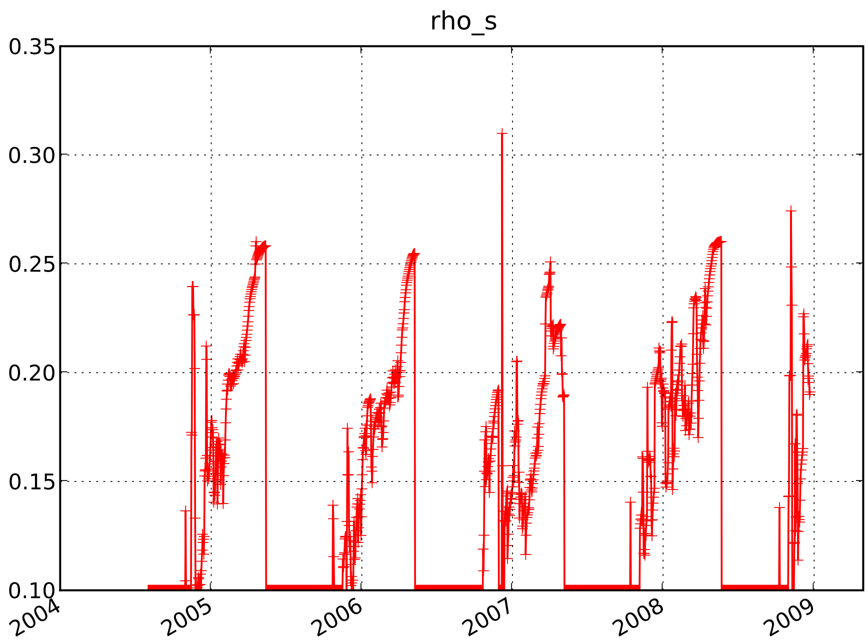

Figure 4: The density of snow in this figure seems to work fairly OK except in winter 2006-2007 when it varies much.

6 Future development

When looking at the results, it seems that in some years the ice remains too shallow. This is due too low snow density from the snow model, which affects to the thermal conductivity of snow to the second (see equation 2). Also, the melting of ice needs better adjustment as the melt parameters in the equations 7 have not been calibrated.

One a more physical calibration parameter can be added. This could be the bottom heat flux Q. It can be added just as a additional term on the right side in equation 5. The bottom heat flux term will be in range of $\approx 0 - 5$W.

The initial ice growth the beginning of the freezing process should be calculated better. Now the ice freezes too fast until the first 10cms of the ice thickness have formed. This is due the boundary condition where the temperature of the air is used as the temperature of the surface. A possible solution is presented in Roeckner et al. (2003), where they state that for numerical reasons, ice formation is suppressed until the cooling, during a time step, becomes large enough to form a slab of ice with thickness $h_i \geq h_{min}$ and $h_{min} = 0.1$m. Similar solution could be used in this model as well. The exact way of implementing this could be determining the degree-day-sum for freezing the first 10cms of ice.

Wind and radiation data can be used to make the model more physical. However, this would require more complex structure and much more initial values. The radiation budget model could be developed in the same way as in Saloranta (2000). With radiation and wind data the surface temperature and heat fluxes could be estimated better.

References

Roeckner, E. et al.: *the atmospheric general circulation model ECHAM5 Part I*, 2003, Max-Planck Institute for Meteorology, Report No. 349, Hamburg

Saloranta, T.M.: *Modeling the evolution of snow, snow ice and ice in the Baltic Sea*, 2000, Tellus, 52A, pages 93-108

Vehviläinen, B., Huttunen, M. and Huttunen, I.: *Hydrological forecasting and real time monitoring in Finland: The watershed simulation and forecasting system (WSFS)*, 2005, Innovation, Advances and Implementation of Flood Forecasting Technology, Confference proceedings, Tromsø,